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The Gravitational Force Generated by an Interaction between Matter and the ZPF field in the Vacuum, and the Property of a Superfluid Vacuum

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Abstract

Jordan-Mbeutchou proposed a model for Newtonian gravity by assuming that space is filled with an ether fluid. But the existence of ether (or aether) in space was denied by the experimental investigation. Contrary to their theory, a new model of gravitation is proposed based on the interaction between matter and the ZPF field in a vacuum. Puthoff proposed a gravitation mechanism that gravitational force is an induced effect associated with ZPF of the vacuum in much the same manner as the van der Waals and Casimir forces. But his theory was not accepted by the science community. Instead of his theory, we consider that space is filled with an electromagnetic fluid consisting of ZPF energy, which is a fluctuation of zero-point energy in the vacuum. From which, it is seen that gravitational force is created by an interaction between the matter and ZPF field in the vacuum. From this theory, the gravitational waves will be radiated as a longitudinal wave which is faster than the light speed. In this context, a toy model of a moving body (or a spacecraft) flowing across a superfluid vacuum presents the possibility to be accelerated beyond any limiting speed and disengaged from friction. This proposed model points to the potentiality to use concepts from a superfluid representation of the universe to devise a process of space propulsion distinct from the ones based on general relativity theory.

Keywords: gravity, ether, ZPF field, electromagnetic fluctuation, gravitational wave, superfluid.

1. Introduction

In 1926 Pascual Jordan ([Born et al., 1926](#)) published the first attempt to quantize the electromagnetic field. In a joint paper with Max Born and Werner Heisenberg, he considered the field inside a cavity as a superposition of quantum harmonic oscillators. In his calculation, he found that in addition to the "thermal energy" of the oscillators there also had to exist infinite zero-point energy term. He was able to obtain the same fluctuation formula that Einstein had obtained in 1909 ([Einstein, 1909](#)). However, Jordan did not think that his infinite zero-point energy term was "real", writing to Einstein that "it is just a quantity of the calculation having no direct physical meaning" ([Mehra, Rechenberg, 2002](#)). Jordan found a way to get rid of the infinite term, publishing a joint work with Pauli in 1928 ([Jordan, Pauli, 1928](#)), performing what has been called "the first infinite subtraction, or renormalization, in quantum field theory" ([Schweber, 1994](#)).

Building on the work of Heisenberg and others Paul Dirac's theory of emission and absorption ([Weinberg, 1977](#)) was the first application of the quantum theory of radiation. Dirac's

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work was seen as crucially important to the emerging field of quantum mechanics; it dealt directly with the process in which "particles" are created: spontaneous emission (Yokoyama, Ujihara, 1995). Dirac described the quantization of the electromagnetic field as an ensemble of harmonic oscillators with the introduction of the concept of creation and annihilation operators of particles. The theory showed that spontaneous emission depends upon the zero-point energy fluctuations of the electromagnetic field to get started. In a process in which a photon is annihilated (absorbed), the photon can be thought of as making a transition into the vacuum state. As early as 1951, P.A.M. Dirac published two papers where he pointed out that we should take into account quantum fluctuations in the flow of the aether (Dirac, 1951, 1952). Inspired by the Dirac ideas, K.P. Sinha, C. Sivaram, and E.C.G. Sudarshan published in 1975 a series of papers that suggested a new model for the aether, in which it is a superfluid state of fermion and anti-fermion pairs, describable by a macroscopic wave function (Sinha et al., 1976; Sinha, Sudarshan, 1978). In their papers, they decided to treat the superfluid as a relativistic matter – by putting it into the stress-energy tensor of the Einstein field equations. This enables us to take an important step – allowing us to describe relativistic gravity as one of the small fluctuations of the superfluid vacuum as well. Based on their ideas, we consider the mechanism of gravitation based on the interaction between matter and the ZPF (zero-point fluctuations) field contrary to Einstein's general relativity theory which claims that the gravitation is due to the curvature of the space.

2. Results

Gravitation from the standpoint of the ZPF energy in the vacuum

Zero-point energy (ZPE) is the lowest possible energy that a quantum mechanical system may have. Unlike in classical mechanics, quantum systems constantly fluctuate in their lowest energy state as described by the Heisenberg uncertainty principle (i.e., the ZPF field). According to quantum field theory, the universe can be thought of not as isolated particles but as continuous fluctuating fields. Puthoff proposed a gravitation mechanism by an interaction between elementary particles and the ZPF field (Puthoff, 1989). He considered that gravitational force is an induced effect associated with ZPF of the vacuum in much the same manner as the van der Waals and Casimir forces.

Jordan-Mbeutchou proposed a model for Newtonian gravity by assuming space is filled with an ether (or aether) fluid (Jordan, Fleury, 2019). Jordan-Mbeutchou modeled gravity as an interaction between matter and ether fluid and assumed that matter can absorb ether fluid proportionally to its mass. This phenomenon can be described by

$$\rho_e \nabla \cdot \vec{v}_e = -\rho / \tau, \quad (1)$$

where ρ_e is a density of ether, \vec{v}_e is its velocity field, ρ is normal matter mass density and τ is a time constant.

Then the radial velocity of the ether which flows in a sphere with a radius r becomes

$$\vec{v}_e = -\frac{1}{4\pi\rho_e\tau} \frac{m(t)}{r^2} \vec{e}_r, \quad (2)$$

where \vec{e}_r is the radial unitary vector.

Hence the amount of the ether momentum which enter the sphere during the time dt is given by

$$\frac{d}{dt} \vec{p}_e = -\int_S \rho_e \vec{v}_e (\vec{v}_e \cdot \vec{n}) dS, \quad (3)$$

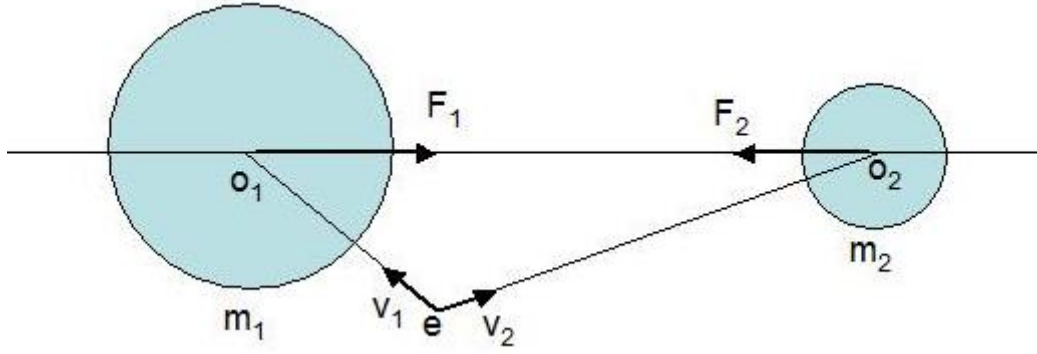


Fig. 1. Two rest masses undergo the force generated the flow of ether

From which, the ether flow is created by the mass m_1 and m_2 at the point o_1 can be estimated by

$$\frac{d}{dt} \vec{p}_e = -\lim_{r_1 \rightarrow 0} \int_S \rho_e (\vec{v}_1 + \vec{v}_2) [(\vec{v}_1 + \vec{v}_2) \cdot \vec{e}_1] dS, \quad (4)$$

where $\vec{e}_1 = \vec{o}_1 e / o_1 e$, and $o_1 e = r_1$.

If we let θ be the angle between \vec{e}_z and \vec{e}_1 , where \vec{e}_z is a vector connecting o_1 and o_2 , it can be obtained as

$$(\vec{v}_1 + \vec{v}_2) \cdot \vec{e}_1 = v_1(r_1) + v_2(r_2) \frac{r_1 - r \cos \theta}{\sqrt{r_1^2 + r^2 - 2r r_1 \cos \theta}}, \quad (5)$$

From which, by integrating the equation (4), the force between two masses can be shown to be

$$F = \frac{d}{dt} \vec{p}_e = \frac{4}{3} \frac{1}{4\pi\rho_e} \frac{m_1 m_2}{\tau^2} \frac{1}{r^2} \vec{e}_z, \quad (6)$$

Thus Jordan-Mbeutchou obtained the Newtonian law of the gravitation from the ether model as

$$F_1 = -F_2 = -\frac{G m_1 m_2}{r^2}, \quad (7)$$

where $G = \frac{1}{3\pi\rho_e \tau^2}$

Contrary to the Puthoff model of gravitation, we consider the gravity mechanism based on the Jordan-Mbeutchou model.

We assume that virtual particles (most of them are virtual photons) created from the ZPF field in a vacuum push matter, then the momentum flux density of virtual particles can be shown as

$$\nabla \cdot \vec{J} = -\rho / \tau_0, \quad (8)$$

where $J = \rho_E / c$ (ρ_E : energy density of the ZPF energy), ρ is a mass density and τ_0 is a retardation time. For the ZPF field, we have like the equation (4) as

$$\frac{d}{dt} \vec{P} = -\lim_{r_1 \rightarrow 0} \int_S (\vec{p}_1 + \vec{p}_2) [(\vec{p}_1 + \vec{p}_2) \cdot \vec{e}_1] / \rho_m \cdot dS, \quad (9)$$

where ρ_m is an equivalent mass density of the ZPF field.

As we can write $\vec{p} = \hbar \vec{\omega} / c$ and $\rho_m = \hbar \omega / c^2$, the amount of momentum created by the ZPF field can be shown as

$$\frac{d}{dt} \vec{P} = -\lim_{r_1 \rightarrow 0} \int_S \frac{\hbar}{c} (\vec{\omega}_1 + \vec{\omega}_2) \frac{c^2}{\hbar \omega} \left[\left(\frac{\hbar \vec{\omega}_1}{c} + \frac{\hbar \vec{\omega}_2}{c} \right) \cdot \vec{e}_1 \right] dS$$

$$= \lim_{r_1 \rightarrow 0} \int_S \frac{\hbar \omega}{c^2} (c \vec{\omega}_1 / \omega + c \vec{\omega}_2 / \omega) [(c \frac{\vec{\omega}_1}{\omega} + c \frac{\vec{\omega}_2}{\omega}) \cdot \vec{e}_1] dS, \quad (10)$$

where $\vec{\omega}_1$ and $\vec{\omega}_2$ are vectors of the radial frequency of the ZPF field at the point e. This equation is equivalent to the equation (4), when we let $c \vec{\omega}_1 / \omega \rightarrow v_1$ and $c \vec{\omega}_2 / \omega \rightarrow v_2$. Then the force at the point o_1 becomes like the equation. (6) shown as

$$F = \frac{d}{dt} P = \frac{4}{3} \frac{1}{4\pi\rho_m} \frac{m_1 m_2}{\tau_0^2} \frac{1}{r^2} \vec{e}_z = \frac{1}{3} \frac{c^2}{\pi \hbar \omega \tau_0^2} \frac{m_1 m_2}{r^2} \vec{e}_z, \quad (11)$$

where a gravitational constant given by $G = \frac{1}{3} \frac{c^2}{\pi \hbar \omega \tau_0^2}$

If we let ω equals to the Plank frequency given by $\omega = \omega_p = 1,855 \times 10^{43} (Hz)$, the retardation time can be estimated as $\tau_0 = 2.7 \times 10^8 (sec)$. As the age of the universe can be estimated by $t_H \approx 4.65 \times 10^{17} (sec)$, then the value of t_H is very close to τ_0^2 , which suggests that the gravitational constant varies with time as claimed by the large number hypothesis.

The equation (11) shows that the gravitational force can be generated by an interaction between matter and the ZPF field in a vacuum. According to this equation, the Newtonian gravitational law can be obtained without ether flow in the vacuum. Hence it is considered that the gravity is an electromagnetic phenomenon induced by the ZPF field in the vacuum and it is not due to the curvature of space as claimed by Einstein.

Gravitational wave propagated as a longitudinal wave

According to the general relativity theory, the gravitational wave can be described by the fundamental equation (Hakim, 1999);

$$h_{\mu\nu} = 16\pi G T_{\mu\nu}, \quad (12)$$

where $T_{\mu\nu}$ is an energy-momentum tensor.

Its retarded solution can be given by

$$h_{\mu\nu}(t, r) \approx \frac{4G}{r} \int d^3 x' T_{\mu\nu}(t - r/c, x'), \quad (13)$$

where r is the distance from the origin.

However, from the standpoint that gravity is induced by an interaction between matter and the ZPF field in the vacuum, gravity wave can be generated by the fluctuation of the electromagnetic field in a ZPF vacuum, and it is radiated as a longitudinal wave into space as a fluctuation of the electric field of virtual particles shown as

$$\varphi(y, t) = \frac{1}{4\pi\epsilon_0} \int d^3 x \frac{\rho_0(x, t - r_{xy}/v_l)}{r_{xy}}, \quad (14)$$

where $\varphi(y, t)$ is a scalar potential of the longitudinal wave at the observation point, ρ_0 is the fluctuation of charge in a ZPF vacuum as shown in Figure 2, r_{xy} is a distance from the origin, and v_l is the speed of the longitudinal wave, which may be faster than the light speed.

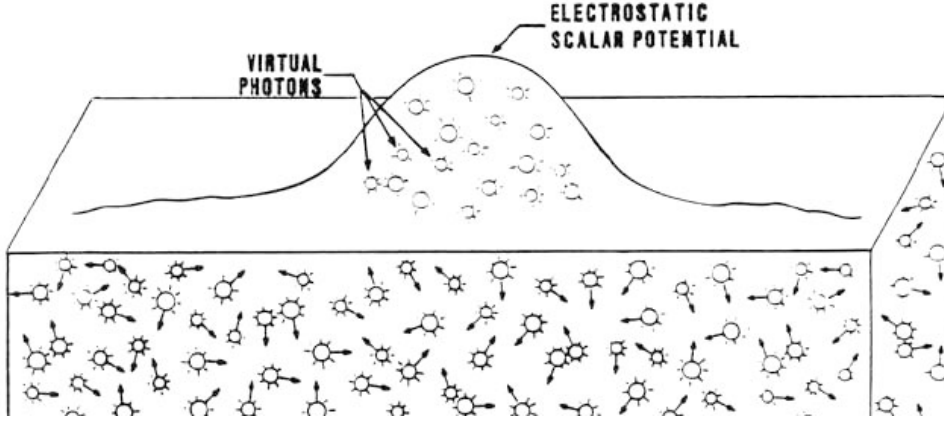


Fig. 2. Gravitational wave generation by ZPF vacuum filled with virtual particles

From this model, gravitational wave can be detected as a fluctuation of electric field in a vacuum.

Material body moving through vacuum superfluid

An alternative theory to the General Theory of Relativity is the Superfluid Vacuum Theory which conceives vacuum as a superfluid performing in some regards to ${}^3\text{He}$ and in some ways a reminiscence of the old aether. We follow the theory developed by Grigory E. Volovik (Volovik, 2003). We shall consider the system aether + matter satisfying the continuity equation

$$m \frac{\partial n}{\partial t} + \vec{\nabla} \cdot \vec{P} = 0 \quad (15)$$

with

$$\vec{P} = mn\vec{v}_s + \vec{P}^M \quad (16)$$

and

$$\vec{P}^M = \int \frac{d^3 p}{(2\pi\hbar)^3} f(\vec{r}, p) \vec{p}. \quad (17)$$

The two-fluid hydrodynamics contains the superfluid vacuum and a normal component, $n = n_n + n_s$. The balance equation.(15) can be written in this context as

$$\frac{\partial n_s}{\partial t} + \vec{\nabla} \cdot \vec{J}_s + \vec{\nabla} \cdot \vec{J}_n = -\frac{\partial n_n}{\partial t} \quad (18)$$

Here, $\vec{J} = \frac{\vec{P}}{m} = n\vec{v}_s + \frac{\vec{P}^M}{m} = \vec{J}_s + \vec{J}_n$. The term $\vec{\nabla} \cdot \vec{J}_s$ represents the flux of superfluid vacuum

entering the material body (or a spacecraft) with boundary $\partial\Omega$. We will assume $\vec{\nabla} \cdot \vec{J}_n = 0$, meaning that there is no matter going out of volume Ω or equivalently, that the amount of matter that leaves Ω equals the amount create inside it, joining an assumption made by Jordan-Mbeutchou' paper (Jordan, Fleury, 2019).

We shall now consider the limit when the counterflow velocity $\vec{w} = \vec{v}_n - \vec{v}_s$ is small. In this situation we may write

$$P_i^M = mn_{nik} w_k = mn_{nik} (v_{nk} - v_{sk}) \quad (19)$$

with n_{nik} representing the normal density tensor defined by

$$n_{nik} = -\sum_p \frac{p_i p_k}{m} \frac{\partial f}{\partial E} \quad (20)$$

where the distribution function is

$$f_{\tau}(\vec{p}, \vec{r}) = \left[\exp \left(\frac{\tilde{E}(\vec{p}, \vec{r}) - \vec{p} \cdot \vec{v}_n(\vec{r})}{k_B T(\vec{r})} \pm 1 \right) \right]^{-1} \quad (21)$$

With $\vec{v}_n(\vec{r})$ denotes the quasiparticle gas velocity and, as usual, the sign (+) is for fermionic quasiparticles in Fermi superfluid, and the sign (−) is for bosonic quasiparticles in Bose superfluid. We could introduce an additional Chern-Simon term into Eq. (19) featuring the effect of spin-orbitronics on the boundary of the moving body forming chiral textures but we will postpone for future work (Dongwook et al., 2017).

The momentum equation now should be written under the form

$$\begin{aligned} \frac{\partial}{\partial t} \iiint_{\Omega} mn v_{si} dv - \frac{\partial}{\partial t} \iiint_{\Omega} m \sum_{\mathbf{p}} \frac{p_i p_k}{m} \frac{\partial f_{\tau}}{\partial \tilde{E}} (v_{nk} - v_{sk}) dv = \iint_{\partial\Omega} \left(m \sum_{\mathbf{p}} \frac{p_i p_k}{m} \frac{\partial f_{\tau}}{\partial \tilde{E}} \cdot \vec{n} \right) dS \\ - \iint_{\partial\Omega} mn (\vec{v}_s \cdot \vec{n}) dS \end{aligned} \quad (22)$$

where Eq. (16) acquires a tensorial character due to Eq. (19) is oriented (to the exterior of the surface) normal tensor. Presently, we necessitate to figure out what would be the total quasiparticles energy created on the surface of the moving body due to its motion along with the physical super vacuum, and, needless to say, due to the extreme complexity, we simply can propose at this stage a toy-model. According to Ref. (Lambert, 1992), a candidate for the energy spectrum, describing so far a qualitative assessment, considering a flow parallel to the z-axis in a pipe, is provided by:

$$\tilde{E}_n = \alpha k_z (v_n - v_s) + \frac{2|k_z|}{L_z} \left[\pi n + \arctg \left(\frac{[\Delta_{\infty}^2 - (E - k_z v)^2]^{1/2}}{E - k_z v} \right) \right]. \quad (23)$$

Here, Δ_{∞} is the bulk energy gap, the parameter α , characterizing the backflow around an arbitrary object, has a value about 2, E is the energy of a quasiparticle for a wave vector \vec{k} , n is an integer ($n = 0, 1, 2, 3, \dots$) and L_z might be identified with the body (or a spacecraft), length, and $v(\vec{r})$ is the local, position dependent superfluid velocity. Observe that in Eq. (22) the first term on in the LHS and the last of the RHS are the ones examined in Ref. (Jordan, Fleury, 2019). We may solve Eq. (22) in the limit of a high rate of quasiparticles emission in which case the last term of Eq. (23) may be discarded leading to an estimation of the surface integral present on the RHS to be of the order of

$$\iint_{\partial\Omega} \sum_{\mathbf{p}} p_i p_k \frac{\partial f_{\tau}}{\partial \tilde{E}} \cdot \vec{n} dS \approx \frac{192 \hbar^5 L_z^3}{\alpha^5} (v_n - v_s)^{-5} \quad (24)$$

We then obtain the differential equation for the variable $w = v_n - v_s$:

$$\frac{\partial w_z}{\partial t} + a w_z - \frac{b}{x^2} + \frac{c}{w_z^5} = 0 \quad (25)$$

Eq. (25) governs the speed and acceleration of a moving body (or a spacecraft) cruising across a superfluid vacuum and it can hold unusual solutions. We had assumed a potential nearby the moving body decreasing as $1/x^2$ perpendicularly to the walls of the moving body.

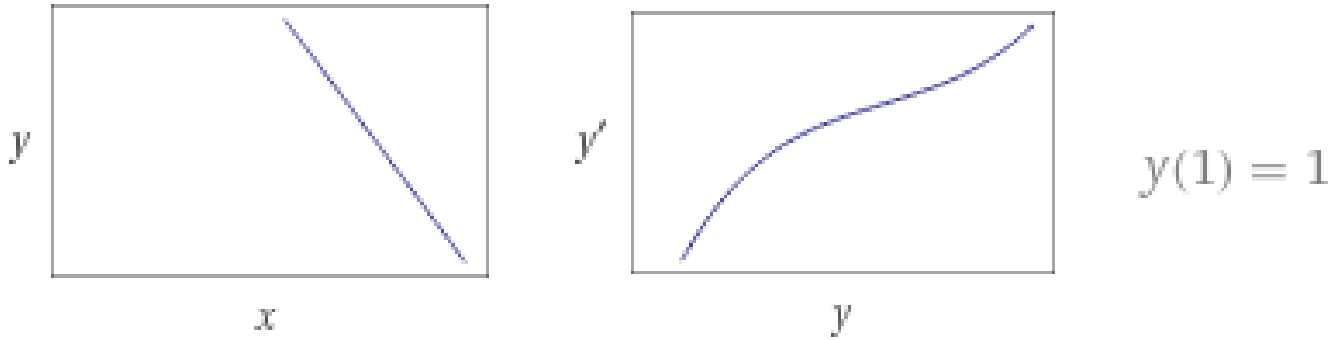


Fig. 3. Numerical solutions for the Eq. (25) with specific parameters ($a = 2, b = 1, c = 0.07$). Left: speed vs. distance from the boundary of the moving body; Right: acceleration vs. speed of the spacecraft crossing the super-vacuum.

Figure 3 reveals the numerical solution for the parameters attesting that with a particular design, the moving body can notice an increasing acceleration when a threshold speed is achieved as it is clear in Figure 3b.

Figure 4 displays the region $x > 0$ to the right of the moving body, racing along the Oz axis, exposing a laminar flow across the boundary. Interestingly, it was recently found that there is no speed limit for an object moving in a superfluid and the reason for the absence of the speed limit is that exotic particles that stick to all surfaces in the superfluid (Auti et al., 2020).

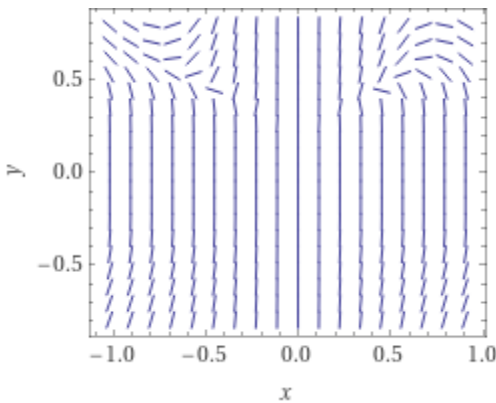


Fig. 4. To the right side of the graphic, it is the region around the moving body showing a recirculation of the quasiparticles fluid in the super-vacuum space but notice that the region near the surface shows no perturbation in the flow

The proposed toy model points to the potentiality to use concepts from a superfluid representation of the universe to devise a process of space propulsion distinct from the ones based on general relativity theory.

3. Discussion

One difficulty in general relativity theory is known as the equivalent paradox. A uniformly accelerated charge is recognized to radiate electromagnetic radiation. However, a charged particle suspended at rest in a uniform gravitational field does not emit radiation. According to general relativity, a uniformly accelerating system in a free space should be equivalent to one at rest in a uniform gravitational field. Hence, in this case, the principle of equivalence seems to be violated.

This problem has been discussed at the classical level without adequate resolution (Boulware, 1980; del Almeda, Saa, 2005). However, the gravity is not due to the curvature of space, but due to the interaction between matter and the ZPF field, this contradiction can be solved.

4. Conclusion

In this article, the gravity mechanism by an interaction between matter and the ZPF field in the vacuum is discussed. According to the Jordan and Mbeutchou model, it becomes clear that the

gravity mechanism can be explained by an interaction between matter and the ZPF field.

From this conclusion, gravitation is not a force to pull but a push from the ZPF field in the vacuum contrary to Einstein's curvature of space. From this standpoint, gravitational waves propagate as longitudinal waves in the vacuum which may be faster than the light speed.

Based on the theory of super-fluid of space, it was found that there is no speed limit for an object moving in a superfluid contrary to Einstein's relativity theory.

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