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Articles

Application of Galileo's Principle of Relativity in the Study of Space Objects with Non-Inertial Motion

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Abstract

Space data processing utilizes geometric methods. This is due to the fact that angular measurements are often used in space research, as stereoscopic photography is impossible to achieve under terrestrial conditions. Geometric construction methods for space conditions are not commonly used on Earth. In space research, the problem of data acquisition is relatively simple. The challenge lies in using space information to process and analyze spatial situations and accumulate experience. A distinctive feature of space research is that imagery is only taken from moving objects (spacecraft), and moving objects are also largely studied. This poses the problem of imagery from moving objects and the problem of geometric constructions taking motion into account. This formulation of the problems necessitates an analysis of the applicability of the principles of relativity to space imaging. This article is devoted to the study of this problem. The principles of relativity, from Galileo, Newton, and Einstein to the present day, consider only inertial systems. The main question is whether the system is inertial or non-inertial. This issue is not addressed in this paper. Instead, the problem of using sensors in non-inertial systems as in inertial systems is investigated. Spacecraft observation systems include a set of different sensors, including a laser rangefinder, a laser scanner, digital cameras, radars, and others. Until recently, laser rangefinders were not used for geometric constructions and determining the coordinates of surface points. They were used only to determine the range or distance to scan points. This article proposes a method for using a laser rangefinder for geometric constructions and obtaining three-dimensional coordinates. This method is applicable to studying the surfaces of planets, other celestial bodies, or other spacecraft encountered during the movement of a research spacecraft. Three spatial simplified situations are presented. Two of these situations demonstrate the possibility of applying the principle of relativity when photographing moving objects or when photographing from moving objects. A simplified method for calculating coordinates using a rangefinder is presented. Two options are considered. The first calculation option uses only the rangefinder and allows for determining two coordinates on the surface of the body in an arbitrary coordinate system tied to the spacecraft trajectory. The coordinates are determined only in the cutting plane. The second calculation option uses a rangefinder and a digital camera. This

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calculation option allows for the determination of three coordinates in a conventional coordinate system associated with the spacecraft's trajectory.

Keywords: space exploration, spacecraft, spatial situation, information perception channel, planetary surface imaging, laser rangefinders.

1. Introduction

Special geometric constructions are often used when studying the surfaces of planets (Barmin et al., 2014; Shaytura, 2020) and comets. Many spacecraft use single-camera imaging (Gospodinov, 2021) to study the surface of a celestial body or the environment in which that celestial body is located. One of the special features of space object research is the study of the surfaces of planets or small celestial bodies (Oznamets, Tsvetkov, 2019). For this reason, spacecraft carry multiple cameras (Savinykh, Tsvetkov, 2001) to obtain more information. Spacecraft observation systems include a variety of sensors, including a laser rangefinder, a long-focus camera with high coordinate accuracy, and a wide-focus camera for surface observation. Angular measurements are often used in space research (Savinykh, 2021) because stereoscopic photography is impossible to achieve under terrestrial conditions. There is a trend in space exploration to use digital cameras (Tagai, Batham, 2024). They can be used for real-time analysis or for sending information to Earth. A digital camera takes a photograph and transmits it via a communication channel to the mission control center. In space exploration, photography is conducted only from moving objects (Bronnikov, 2023). Moreover, most space objects are also moving. This situation poses the problem of photographing from moving objects, the problem of geometric constructions taking into account motion, and the problem of processing such constructions. This formulation of problems leads to the need to analyze the application of the principles of relativity to space photography. One of the first principles of relativity belongs to Galileo. It states that all laws of mechanics are the same (invariant) in all inertial reference frames. Inertial reference frames are those that are either at rest or moving uniformly and rectilinearly relative to each other. In space, there is no rectilinear motion (Savinych, 2017, Kudzh, 2022). It has curvature, and the shortest distance is not a straight line, but a geodesic line. Of interest is the application of survey methods in the absence of inertial motion conditions. In this case, the goal is not to determine whether the system is at rest or moving at a constant velocity. The goal is not to determine inertiality, but to process measurements from such a system as an inertial reference frame.

2. Results and discussion

Geometric Constructions in the Study of Certain Situations

Consider the three spatial situations depicted in Figures 1, 2, and 3. We introduce the concepts of spacecraft (SC) and "flyby body" (BO). A flyby body can be a planet or another celestial body, including another cylindrical spacecraft.

Points of the spacecraft's trajectory are denoted by T. Points on the surface of the flyby body are denoted by P. Point Q denotes the projection point. In situations 1 and 3, this is the projection onto the surface of the flyby body from the spacecraft's trajectory. In situation 2, this is the projection onto the spacecraft's trajectory from the surface of the flyby body.

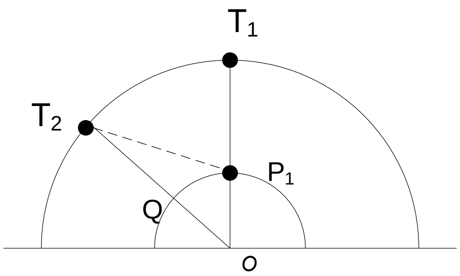

Fig. 1. The motion of the spacecraft relative to a non-rotating object

Situation 1 corresponds to the case where the spacecraft moves relative to a non-rotating object along a circular or elliptical trajectory. During observation time t1, the spacecraft moves along a trajectory from point T1 to point T2. Initially, point T1 in the spacecraft's orbit corresponds to point P1 on the surface of the flyby body. It can be assumed that the spacecraft rotates relative to a certain center O (Fig. 1) with an angular velocity ω1. The radius of rotation is determined by the formula.

R1=OT1= OT2

The distance traveled D along the trajectory is determined by the formula

D=R1 t1 ω1

In addition, the spacecraft's inertial system determines the relative coordinates at observation points T1 (X1, Y1, Z1) and T2 (X2, Y2, Z2). This is the conventional (relative) coordinate system of the spacecraft's trajectory, tied to these orbital points. Point P1 on the flyby body's surface is the nadir point relative to the spacecraft at point T1. Point Q on the flyby body's surface is the nadir point relative to the spacecraft at point T2. Point T1 on the trajectory (SC) is the zenith point relative to the flyby body's surface. The coordinates of point P1 are unknown. However, using rangefinding devices, the distances L1 and L2 can be determined (Figure 1).

L1= T1P1

L2=T2P1

Situation 2 (Figure 2) corresponds to the case where the spacecraft is hovering in space at point T1, while the flyby body rotates. Initially, point P1 on the surface of the flyby body is the nadir point relative to the spacecraft at point T1.

To illustrate the relativity of this situation, we choose the same observation time t1. During this time, the point on the surface of the flyby body moves in space from point P1 to point P2. This point does not move on the surface of the body, but it moves in space. The spatial coordinates of point P2 (u2, v2, l2) change relative to the spatial coordinates of point P1 (u1, v1, l1).

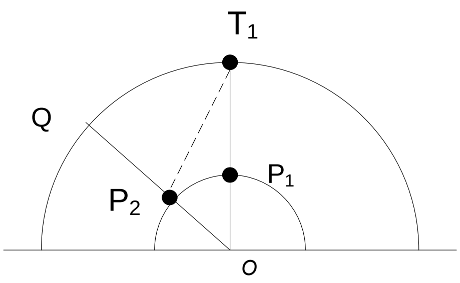

Fig. 2. Rotation of an object relative to a hovering spacecraft

The position of point T1 remains constant during observation time t1. Using rangefinding devices, distances L1 and L3 can be determined (Figure 2).

L1= T1P1

L3=T1P2

To emphasize the relativity of the processes, we assume that the flyby body rotates with the same angular velocity ω1. The distance traveled by a point on the surface of the flyby body, D2, is determined by the formula

D2=R2 t1 ω1

The radius R2 on the surface of the flyby body is determined by the formula

R2=OP1= OP2

Point P1 on the flyby body's surface is the nadir point relative to the spacecraft at point T1. Point Q on the spacecraft's trajectory is the zenith point relative to point P2 on the flyby body's surface. Point Q in situation 2 (Figure 2) corresponds to point T2 in situation 1 (Figure 1). Point Q in situation 1 (Figure 1) corresponds to point P2 in situation 2 (Figure 2). This comparison proves the relativity of position in space for special cases of non-inertial motion.

In practice, the spacecraft is most often moving and the flyby body is rotating. This situation is shown in Figure 3. They rotate at different angular velocitie ω1 и ω2.

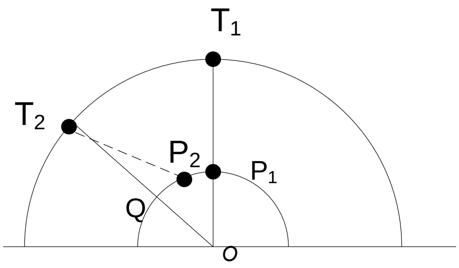

Fig. 3. Rotation of an object and motion of a spacecraft

We'll assume the observation time is t1. The distance traveled along the spacecraft's trajectory, D1, is determined by the formula

$$D1=R1\ t1\ \omega 1$$

The path traveled by a body D2 on the surface of a body during its flight in space is determined by the formula

$$D2=R2\ t1\ \omega 2$$

Coordinate calculations.

We will show the coordinate calculations for situation 1 in Fig. 1. Consider the triangle T1, T2, P1 (Fig. 1). For simplicity, we will introduce the following notations:

T1T2=a; T1P1=b; T2P2=c.

The basic idea is that the area of a triangle can be calculated from the lengths of its sides, as well as its height and base. According to Heron's formula, if the lengths of the sides are known, the area of the triangle S is calculated using the formula

$$S = \sqrt{p(p-a)(p-b)(p-c)}\ (1)$$

In expression (1), a, b, c are the lengths of the sides of the triangle, and p is the semiperimeter, which is calculated as: p = (a + b + c)/2

Let's introduce the auxiliary construction shown in Figure 4.

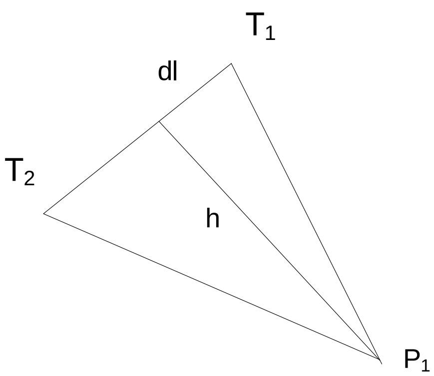

Fig. 4. Estimating the area of triangle T1T2P1

From point P1, draw a height to side T1T2 and denote it as h. The area of triangle T1T2P1 can also be determined using the formula

$$S= (a\ h)/2\ (2)$$

Let's define a relative coordinate system with the origin at point T1. In a relative coordinate system, the direction T1T2 can be chosen as the X-axis. The normal to it defines the Y-axis. This normal direction coincides with the direction of the triangle's altitude h. The value of h determines the YY coordinate of point P1 in a relative coordinate system with the origin at point T1. The segment dl on the line T1T2 determines the X-coordinate of point P1.

Let's equate the left-hand sides of equations (1) and (2) and eliminate the square root.

$$½\ a\ h = [p\ (p-a)\ (p-b)\ (p-c)]^2\ (3)$$

We calculate h

$$h= 2 [p (p-a) (p-b) (p-c)]^2 /a \quad (4)$$

Expression (4) defines the YY coordinate of point P1 in the relative coordinate system, in the cutting plane tied to the spacecraft trajectory. The value of dl on the line T1T2 defines the X coordinate of point P1 in the relative coordinate system.

$$dl = \sqrt{b^2 - h^2} \quad (5)$$

Thus, two coordinates of point P1 are determined in the plane measuring the distance to point P1. The third coordinate can be calculated using a photograph or digital image. It should be noted that the YY coordinate of point P1 is measured in the cutting plane. Essentially, it is a slant range, not a coordinate.

Figure 5 illustrates the use of an image to calculate three "correct" coordinates. The figure shows the slant range YY and the Y coordinate in a three-dimensional coordinate system. The segment dl corresponds to the X coordinate.

$$YY = \sqrt{Y^2 + Z^2} \quad (6)$$

The coordinates of the image of the point are determined from the photograph

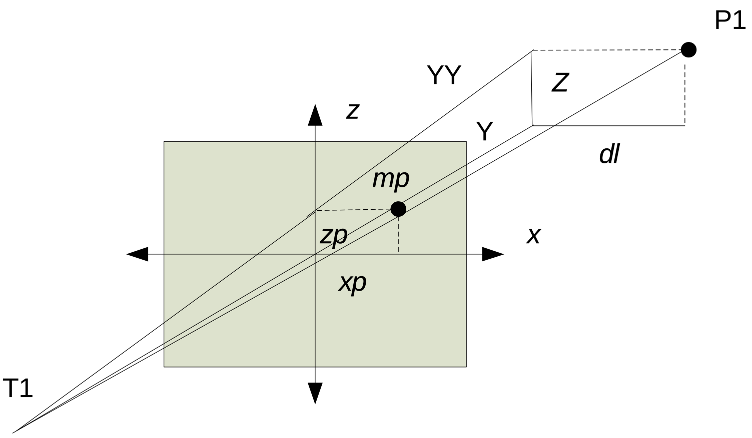

Fig. 5. Determining coordinates using an image.

The coordinates of point P1 are in the relative coordinate system (X, Y, Z). The unknown coordinates are (Y, Z). The xp coordinate in the image is used to determine the scale M between the image and reality.

$$M=dl/xp \quad (7)$$

The Z coordinate is determined from the zp coordinate in the image

$$Z= M\, zp= zp\, dl/xp \quad (8)$$

The Y coordinate is determined from expression (6)

$$Y = \sqrt{YY^2 - Z^2} \quad (9)$$

Calculations for more complex situations will follow in future articles.

3. Conclusion

Some types of non-inertial motion obey the principles of relativity. For space exploration, the inertia of a system is not as important as the ability to transfer moving imagery conditions to stationary ones. This study demonstrates the relativity of position in space for specific cases of non-inertial motion. The principle of relativity can be applied to non-inertial systems when the parameters of moving bodies are recorded from them. The principle of relativity can be used to transfer a coordinate system to different points on a moving object.

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